

Rapidity dependence in holographic heavy ion collisions

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Abstract

We present an attempt to closely mimic the initial stage of heavy ion collisions within holography, assuming a decoupling of longitudinal and transverse dynamics in the very early stage. We subsequently evolve the obtained initial state using state-of-the-art hydrodynamic simulations, and compare results to experimental data. We present results for charged hadron pseudo-rapidity spectra and directed and elliptic flow as functions of pseudo-rapidity for $\sqrt{s_{NN}} = 200$ GeV Au-Au and 2.76 TeV Pb-Pb collisions. The directed flow interestingly turns out to be quite sensitive to the viscosity. The results can explain qualitative features of the collisions, but the rapidity spectra in our current model is narrower than the experimental data.

1. Introduction. Collisions of relativistic heavy ions have been understood to quickly form a quark-gluon plasma (QGP), which thereafter evolves according to relativistic hydrodynamics with small viscosity [1–3]. To make this paradigm more precise it is of crucial importance to understand the early time far-from-equilibrium stage and be able to accurately compute the initial state of the hydrodynamic plasma. This is particularly challenging because of the generally non-perturbative nature of Quantum Chromo Dynamics (QCD). In this Letter we model this initial stage at strong coupling using holography. We then provide the resulting energy density and flow velocity distributions as input for the subsequent viscous hydrodynamic evolution.

This first study makes several strong assumptions. In particular, we describe the entire early stage of the collision in the strong coupling limit. Further, we work in the canonical holographic theory, $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory with a large number of colors. This conformal theory has no confinement or asymptotic freedom and hence is very different from QCD. Nevertheless, at energy scales relevant for the early stage of heavy ion collisions the theories are more similar, which is where we use the SYM theory as an approximation for QCD. Lastly, this study neglects any chemical potentials and event-by-event fluctuations.

Previous studies of QGP thermalization at strong coupling notably include homogeneous [4] and boost-invariant [5, 6] settings, less trivially the collisions of planar shock waves [7–9] and transverse expansion [10–13]. It is the purpose of this Letter to combine lessons from these works to construct the initial conditions for hydrodynamics, in particular using the fast thermalization [5, 6], a universal rapidity profile [8, 9] and a simple formula for transverse flow [10, 12]. We thereafter employ the MUSIC viscous relativistic hydrodynamic simulation [14–16] to evolve this profile till freeze-out, after which we obtain the particle spectra using the Cooper-Frye formalism [17].

At strong coupling there exists a specific rapidity profile which is notably different from other approaches [9, 18], and we therefore focus on the rapidity depen-

dence of observables, in particular directed flow, which is non-trivial to reproduce with initial conditions that model the longitudinal structure [19]. Without including full event-by-event fluctuating initial conditions, it is well-known that quantitative agreement with experimental data cannot be achieved. Nevertheless, comparing our results to experimental data for the pseudo-rapidity distributions of charged hadrons, we find that while the profiles are narrower than the experimental data, agreement is better than expected given the very narrow initial rapidity profile characteristic of holography [8]. Also, we find good quantitative agreement of the directed flow as a function of pseudo-rapidity, both at RHIC and LHC energies around mid-rapidity. A more complete analysis will therefore be of great interest, as will be a direct quantitative comparison with models inspired by perturbative QCD [20].

Lastly, we found two important results, likely independent of the holographic framework used here. Firstly, we found that almost half of the produced entropy is due to viscous entropy production, even though the viscosity is small. Secondly, the rapidity profile of the directed flow is very sensitive to the viscosity, and may as such be useful to improve future estimates of the viscosity of the QGP.

2. The initial state from holography. Holography provides a precise mapping between certain strongly coupled quantum field theories and gravitational theories with one extra dimension. Here we will use the original and simplest example, where strongly coupled $SU(N_c)$ $\mathcal{N} = 4$ super Yang-Mills theory in 3+1 dimensions is mapped to a gravitational theory in 4+1 dimensional anti-de-Sitter spacetime. As has often been done to describe heavy ion collisions, the individual ions are modeled by gravitational shock waves, which in the Yang-Mills theory correspond to lumps of energy moving unperturbed at the speed of light.

An important assumption we will be making is the decoupling of the longitudinal dynamics and the transverse dynamics in the stage before we use hydrodynamics, which we here take to be the first 0.1 (0.2) fm/c of the collision at LHC (RHIC). This assumption should hold

to high accuracy as long as the typical transverse structures are larger than this time. For an average energy density this would even be the case in systems as small as the ones in p-A collisions, but note that it prevents us from considering event-by-event fluctuations smaller than this length. This assumption, however, allows us to split up the holographic calculation into a longitudinal one with translational symmetry in the transverse plane, and a transverse calculation with boost invariance.

The longitudinal dynamics has been studied in [7, 9, 18, 21–23] which led to two main lessons: the plasma thermalizes very fast in the sense that *viscous* hydrodynamics becomes applicable in times perhaps as short as 0.05 fm/c at LHC energies. Furthermore, at LHC energies the temperature is universally approximately constant in the z (beam) direction at constant time [24] (Fig. 1), where importantly the time is determined in the local center of mass frame (LCOM) of the local transverse energy densities [9]. This means it does *not* refer to the nucleon-nucleon center of mass. This observation is also valid for asymmetric longitudinal profiles [9], which is relevant for off-central collisions. Remarkably, even though this temperature profile is not boost invariant at all, the longitudinal velocity profile shows approximate Bjorken behavior: $v_z = z/t$.

For the transverse energy densities ϵ_L and ϵ_R of the left- and right-moving nuclei we will take an integrated Wood-Saxon distribution [25]. In matching the holographic computation to hydrodynamics we then use the following formula for the energy density [26]:

$$\mathcal{E}(t) = \frac{N_c^2 \Lambda^4}{2\pi^2} \left[\frac{1}{(\Lambda t)^{4/3}} - \frac{2\eta_0}{(\Lambda t)^2} \right], \quad (1)$$

where $\eta_0 = \frac{1}{\sqrt{2}^{33/4}}$, we take $N_c = 1.8$ such that the EOS matches lattice data ($e/T^4 \approx 12$) [27, 28] and Λ has to be extracted from Fig. 1 numerically as $\Lambda = 0.37 \epsilon^{1/3}$ [29], with $\epsilon = \sqrt{\epsilon_L \epsilon_R}$ the center of mass energy density per transverse area of the collision (all quantities depend on x_\perp). Eq. (1) was originally found as a solution to first order hydrodynamics in a boost invariant context, but it turns out that the formula also works well to describe the energy density of our (non-boost invariant) collision at midrapidity [30]. There is no z dependence since the local energy density is approximately constant at constant t (Fig. 1) and the dependence on the transverse coordinates is entirely contained in the transverse energy densities. The rapidity profile then follows by converting proper time and rapidity to normal time, where as described above we have to boost to the local center of mass frame. We hence use $t = \tau \cosh(y + \delta y)$, with y the spacetime rapidity and $\delta y = \frac{1}{2} \log(\epsilon_L/\epsilon_R)$ the shift to the LCOM [9].

When the longitudinal width of the incoming nuclei is large enough, the high energy regime plotted in Fig. 1 is no longer applicable [8, 31]. For RHIC energies this is

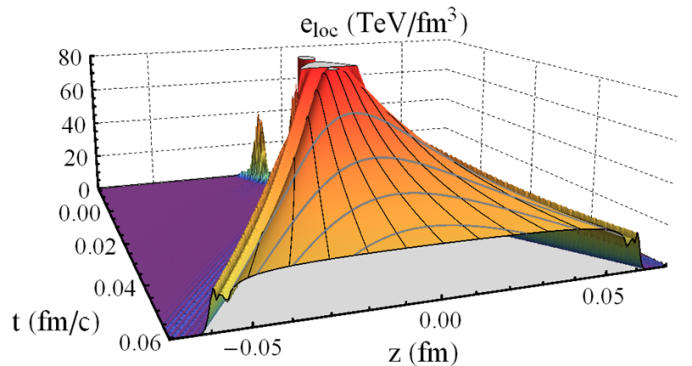


Figure 1. We show the local energy density as a function of time and the longitudinal direction, in the center of mass frame. Units are such that the energy per transverse area of the initial shock matches the center of a LHC Pb-Pb collision. Note that this frame will depend on the position in the transverse plane. The black and grey curves denote stream lines and constant proper time curves.

a significant effect, and it was found that the rapidity profile is approximately 30% narrower and 83% higher as compared to the high energy regime described above [8]. We incorporated this by a simple rescaling of the hydrodynamic initial data.

By causality the transverse profile of the energy density does not change much in this very early stage. A non-trivial transverse flow does develop [32], which has been studied holographically in [10, 11]. These works found that the transverse fluid velocity is proportional to the transverse gradient of the energy density, with a numerical coefficient extracted in [12]:

$$v_i = -0.33 \tau (\partial_i e)/e, \quad (2)$$

with $i = x$ or y , τ the proper time, and e the local energy density, now depending on all spatial coordinates.

We can now proceed to the construction of the complete initial condition. Given two colliding objects, having their respective initial energy profiles ϵ_L and ϵ_R as a function of the transverse plane, we construct for all transverse coordinates the relevant rapidity and energy of the LCOM. We then map the holographic longitudinal profile (using Eq. (1)) to obtain the energy density as a function of rapidity at a fixed initial proper time $\tau_{\text{ini}} = 0.1$ (0.2) fm/c at LHC (RHIC). Once we have the local energy density we obtain the local transverse velocity from Eq. (2).

In the approximation described above holography provides the complete energy density, in principle only requiring the transverse energy densities. Nevertheless, we found that this approach overestimated the total multiplicity, possibly because of not including fluctuations and the assumption of very strong coupling. For this reason we divided the energy density by a factor of 20 (6) for the top LHC (RHIC) energies so that charged hadron multi-

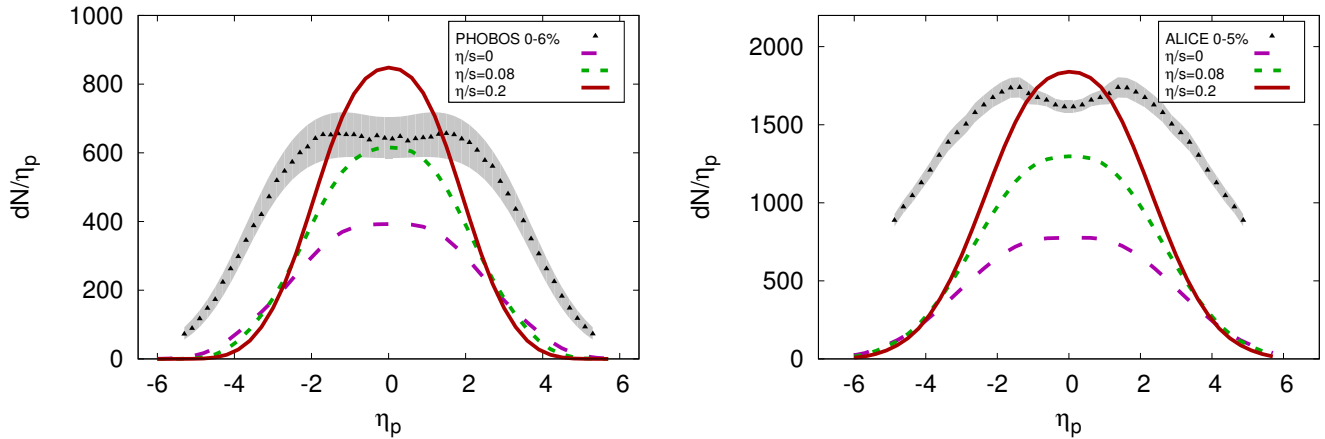


Figure 2. Charged hadron multiplicity for top RHIC (200 GeV) (left) and LHC (2.76 TeV) (right) energies as a function of pseudo-rapidity η_p for a collision with impact parameter 1 fm with $\eta/s = 0, 0.08$ and 0.2 . In all cases the spectrum is narrower than the experimental data from the PHOBOS [33] and ALICE [34] collaborations, respectively. However, the spectrum is much wider than the initial state rapidity profile obtained from just holography (see Fig. 1 and [8]).

plicities would be close to the experimental data, which we will address again in the discussion.

Having obtained the energy density and the local fluid velocity we proceed by using them to initialize the relativistic viscous fluid dynamic simulation MUSIC [14–16]. We set the initial viscous stress tensor to zero and will use various constant values of the shear viscosity to entropy density ratio η/s .

For the equation of state we use the parametrization “s95p-v1” from [35], obtained from interpolating between lattice data and a hadron resonance gas.

At a constant freeze-out temperature of 150 MeV the fluid is converted to particles using the Cooper-Frye algorithm [17], after which resonance decays are computed, including all resonances of energy 2 GeV or less.

3. Results and discussion. We present the resulting charged hadron spectra as a function of pseudo-rapidity in Fig. 2 for top RHIC ($\sqrt{s} = 200$ GeV) and LHC ($\sqrt{s} = 2760$ GeV) energy collisions. We used an impact parameter of 1 fm to simulate central collisions. The rapidity spectrum for RHIC and LHC collisions are too narrow, but the relative increase in width from RHIC to LHC is similar to the data. Also, it is noteworthy that the rapidity spectra found here are much wider than the holographic initial profile of width 0.9 found in [8], stressing the importance of hydrodynamic evolution.

We note that the previously observed [16] effect of viscosity, namely the reduction of the effective longitudinal pressure and correspondingly smaller multiplicities at larger rapidity are also visible in Fig. 2.

One of the main motivations of this work was to study the rapidity dependence of the directed flow, which we show in Fig. 3 and Fig. 4 for collisions at RHIC (200 GeV) and LHC (2760 GeV), respectively. Here we used an im-

pact parameter of 8 fm to simulate approximately 35% central collisions.

We have computed the event plane ψ_1 at forward rapidities and then evaluated

$$v_1 = \langle \cos(\phi - \psi_1) \rangle, \quad (3)$$

where ϕ is the azimuthal angle of charged hadrons in momentum space and the average $\langle \dots \rangle$ is over the particle momentum distributions.

Interestingly, this quantity turns out to be very sensitive to the η/s ratio, and can as such possibly be a good probe of the viscosity. The shape of the curve matches the STAR result quantitatively close to midrapidity, which can be seen as a partial success of the combination of the holographic rapidity profile and the LCOM description presented above. At larger rapidities, one should not trust the results for v_1 , because the multiplicity distribution is not described well in this regime. The bands shown in Figs. 3 and 4 describe uncertainties from inaccuracies in the determination of the freeze-out surface. They are obtained by mirroring the result around the origin, which is a symmetry of the hydrodynamic initial condition.

Fig. 5 shows the elliptic flow as a function of pseudo-rapidity, compared to experimental data from the CMS collaboration [40]. Similar to the result for multiplicity vs. pseudo-rapidity (Fig. 2), the pseudo-rapidity dependence of v_2 is stronger than in the experimental data, while the overall magnitude is close to the experimental data when using $b = 8$ fm and $\eta/s = 0.2$.

We wish to stress that the presented model is very constrained compared to competing models, and basically has no free parameters apart from the energy density rescaling. The only input for the initial stage is the radius and energy of the incoming nuclei and the equation

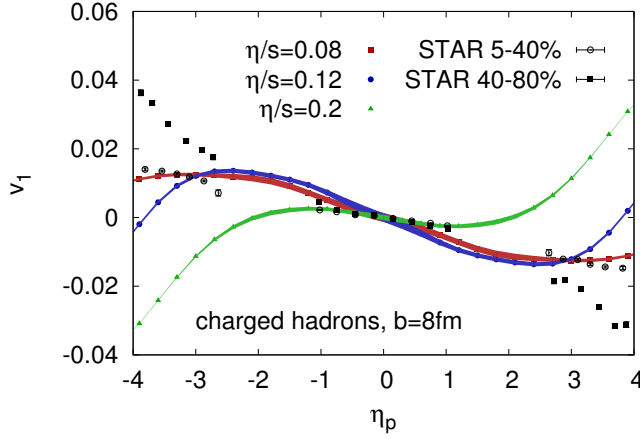


Figure 3. Directed flow v_1 as a function of pseudo-rapidity η_p for 200 GeV collisions at RHIC with $b = 8$ fm (approximately 35% centrality) using $\eta/s = 0.08, 0.12$, and 0.2 . While a direct comparison with experimental data is difficult without a proper event-by-event analysis, the shape around midrapidity matches STAR data for 5-40% and 40-80% centrality classes [36] quantitatively around mid-rapidity. The results for large rapidity are uncertain, because there $dN/d\eta_p$ is in disagreement (Fig. 2). This observable is quite sensitive to the viscosity, which is interesting in its own right. The bands describe uncertainties in v_1 stemming from numerical inaccuracies in the freeze-out surface finding.

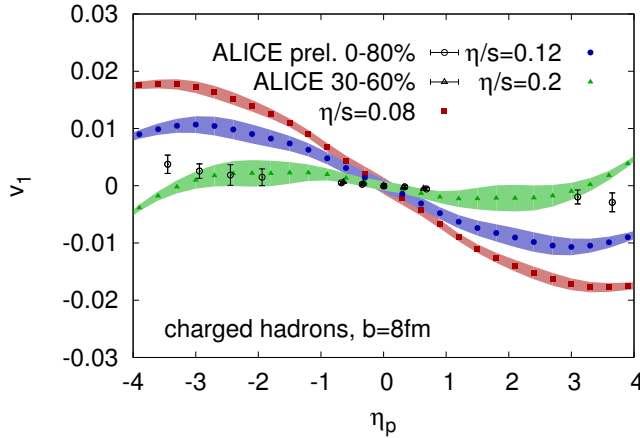


Figure 4. Directed flow v_1 as a function of pseudo-rapidity η_p for 2.76 TeV collisions at LHC with $b = 8$ fm using $\eta/s = 0.08, 0.12$, and 0.2 . While a direct comparison with experimental data is difficult without a proper event-by-event analysis, the shape around midrapidity matches preliminary ALICE data for the 0-80% centrality class [37, 38] and published data for 30-60% centrality [39] quantitatively. The bands describe uncertainties in v_1 stemming from numerical inaccuracies in the freeze-out surface finding.

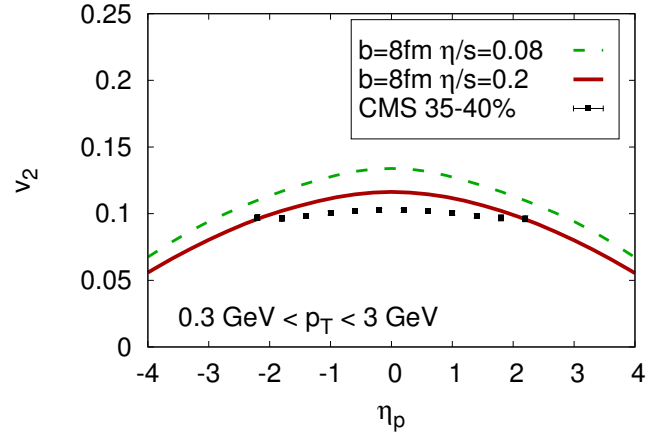


Figure 5. Elliptic flow v_2 as a function of pseudo-rapidity η_p for 35-40% central collisions at 2.76 TeV compared to experimental data from the CMS collaboration [40]. We compare two different values of η/s , 0.08 and 0.2.

of state of QCD ($e/T^4 \approx 12$). Note, however, that adding a more refined transverse energy profile with fluctuations in the future will introduce extra scales relating to those fluctuations.

Nevertheless, when directly comparing with data our simple model has two clear problems. Firstly, to get a reasonably $dN/d\eta_p$ at mid-rapidity we artificially reduced the initial energy density by a factor 20 (6) for LHC (RHIC) collisions, which are large factors. Secondly, the $dN/d\eta_p$ spectrum is still too narrow, even though the width is much more in line with experimental data than the initial strong coupling profile found in [8]. These problems can be partly attributed to the fact that QCD has an intermediate coupling strength, which will intuitively lead to more particles at higher rapidity than in the strong coupling limit presented here. Also, it may be possible that holography is better thought of as providing a description of the (soft) gluons, which carry only part of the energy of the nucleus. The valence quarks perhaps require a different picture [41]. Furthermore, importantly, including fluctuations also partly resolves both problems. The reason is simple: fluctuations will cause QGP to end up at high positive or negative rapidities, thereby widening the rapidity profile. This widened rapidity profile will also give a lower total multiplicity, since the total input energy is fixed and particles at higher rapidity carry much more energy.

Lastly, this work opens up many possibilities for further research. First of all the holographic model can be improved to model heavy ion collisions more realistically. Including finite coupling corrections will be an important step in this direction (see i.e. [42]). It will also be essential to include event-by-event fluctuations. Interestingly, these studies will rely on a particularly rich set

of physics, from numerical general relativity to relativistic hydrodynamics to particle decays, which will allow to make a quantitative comparison of results obtained using holography to part of the experimental data at RHIC and LHC.

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